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Student Roles and Mathematical Competence in Two Contrasting Elementary Classes

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This article focuses on a group of elementary students and their mathematics experiences in two classes. The two classes featured mathematics lessons that were organised in contrasting ways. The purpose of this article is to examine students' views of their experiences in these different mathematics classes. Through a focus on identity and competence, the analysis presents insights in relation to how students make sense and talk about what competence entailed in the different classes of which they are members.

A number of attempts have been made to describe mathematics classrooms that emphasise understanding and inquiry and are at the same time consistent with reform recommendations (e.g., Ball, 1993; Brown, Collins, & Duguid, 1989; Lampert, 1990; Cobb, McNeal, Yackel, & Wood, 1992; National Council of Teachers of Mathematics, 1989, 1991, 2000; Richards, 1991). Such descriptions have collectively emphasised a shift to mathematics classrooms as learning communities rather than merely a collection of individuals. In these reform-aligned and inquiry-based classrooms, discussions focus on students' thinking and attempt to unpack relationships among quantities and concepts in relation to different algorithms. Participation structures in such classrooms are more complex with students posing questions, providing justifications, making conjectures, and offering alternative explanations (Ball, 1993; Ball & Cohen, 1999; Lampert, 1990). Students, in such classrooms, are expected to explain their thinking and interpretations as they solve mathematical tasks and justification rests heavily on explaining one's reasoning and the rationales for procedures. Clearly, reform recommendations have called for dramatic changes that have implications for all aspects of mathematics classrooms.

One of these implications is a shift in students' roles in mathematics classrooms. Reform recommendations require students to engage in learning in different ways, as compared to their roles with more traditional instruction. Some argue that being successful in reform-based, inquiry classrooms demands different forms of knowledge and of participation to which some students have access and others do not (Lubienski, 2000; Zevenbergen, 1996). The multiple communities of which students are members, including the classroom, have implications for students' access to these different ways of participating. At the same time, others indicate the potential of reform practices in addressing issues of differential achievement and persistence in mathematics learning (Boaler, 1997). With this continuing debate, many in mathematics education have emphasised the need for examining the ongoing consequences of reform recommendations for students' learning of mathematical ideas and development

of mathematical dispositions (Boaler, 2002; Lubienski, 2002), and at such a level of detail to inform practices that are specific to mathematics teaching and learning (Boaler, 2002). Such analyses would examine how reform practices play out in interaction in classrooms, and their implications for particular kinds of learners. Analyses of this type would be in contrast to studies in which different teaching approaches are assessed solely in light of students' reasoning on particular tasks (Bransford, Brown, & Cocking, 2000).

One consequence that is worthy of investigation relates to how students view their roles within reform classrooms and their ideas about the *learning practices* (Cohen & Ball, 2000) in which they engage. These learning practices are a part of what it means to be a *competent mathematics student* in a particular class. This image for any given class develops over time and includes the expectations and obligations that are wrapped up in students' roles as mathematics students in particular classroom contexts. The purpose of this article is to examine one group of elementary students' understandings of their roles in two classes, one during their first grade year and the second during their second grade year, in which mathematics instruction was organised in different ways. The analysis reported in this article emphasises the importance of listening to students — not just their mathematical thinking and reasoning — but what they are thinking as they engage in certain activities, as they *do* certain mathematics as it is constituted in specific classrooms. Prior to presenting students' comments about their roles, I begin with a discussion of the ideas that are central to the analysis, that of identity and competence.

Identity and Competence in the Mathematics Classroom

It is generally agreed upon that both the micro processes of the classroom as well as the broader social and cultural contexts in which mathematics learning is situated contribute to students' relationships with mathematics as it is constituted within classrooms and across classrooms over time (Martin, 2000; Nasir & Hand, 2006). Such micro and macro processes draw attention to the multiple communities of which students are members including those of the classroom, school, family, neighborhoods, and broader discourse communities.

Clearly, the mathematics classroom is a community of considerable interest when considering students' developing relationships with mathematics. I view the classroom as a space in which resources interact to provide certain kinds of opportunities for students (Cohen & Lotan, 1995; Engle & Conant, 2002; McDermott, Goldman, & Varenne, 2006). This view of the classroom shifts the focus from students to students' interaction with aspects of the classroom. Difficulties that are typically viewed as deficiencies to be remediated in students are deemed as constructions that arise from the interaction among individuals, resources, and the arrangement of the classroom context. The classroom context may include situations to develop a deep understanding of, and a sense of appreciation, for mathematical ideas. In addition, these opportunities may present situations in which students can display their competence. Mathematical competence within this classroom space can be seen as constructed by, and

through, different resources. Therefore, what counts as mathematical competence differs from classroom to classroom. What becomes constructed as mathematical competence, the roles of the teacher and students, and other features of the classroom can be seen as contributing to students' developing relationships with mathematics. However, aspects outside of the classroom contribute to students' identities in relation to mathematics (Martin, 2000; Walker, 2006). We consider these aspects as we examine the literature on identity from a socially situated perspective.

I first draw on the work of Wenger (1998) to clarify central aspects of identity as socially constructed. Wenger, in his book *Communities of Practice*, describes identity as involving mutual engagement and concrete relations among individuals. His concept involves identity as constructed by individuals as they engage in the day-to-day activities of a community of practice. This approach appears more useful to my purpose in this analysis, which focuses on students' roles in different classroom communities. From Wenger's perspective, as one participates in a community, one can be seen to learn the ways of thinking and acting that are valued within that particular community. Much of this learning is implicit and involves unarticulated suppositions and assumptions as well as particular ways of speaking and using artifacts. This learning also includes students' development of a sense of who they are as people within that community. In other words, students become who they are in a class by being able to play a part in the relations of engagement that constitute it (Wenger, 1998). From this perspective, students' emerging identities are viewed as forms of individuality that are defined with respect to the types of competence that membership in the classroom entails. Therefore, a focus on identity as socially situated emphasises the relationships between individual sense making and "collective" community practices.

Within mathematics education, a growing number of studies have focused on identity in order to understand issues of motivation, affiliation, and persistence (Boaler & Greeno, 2000; Cobb & Hodge, 2007; Martin, 2000; Sfard & Prusak, 2005). Martin's (2000) influential study focusing on mathematics identity spans the different levels of the classroom, school, local community, and broader community to investigate aspects that contribute to identity development. Through his work of understanding African American students' success and failure in mathematics, he has developed a framework for investigating mathematics identity. Initially in his study, Martin focused on the role of the teacher, instructional practices, and curriculum in understanding students' developing relationships with mathematics. As he sought answers to what he was observing in school and mathematics classrooms, he was compelled to examine the broader discourses and communities in which students participated. The *sociohistorical* level of his framework focuses on historically-based practices that have discriminated against African Americans and have prevented many African Americans' substantial participation in mathematics. The *community* level of Martin's framework documents how the collective history of African Americans plays out in individuals' narratives of their personal struggles. As Martin writes:

> African-American parents and community members respond to their experiences in ways that send implicit and explicit messages — positive and negative — about the importance of mathematics learning and knowledge to their children. (p. 38)

These messages that Martin describes contribute to how young people come to think about the events that they experience in mathematics and science classes. This framework examines *the school* and classroom level by focusing on the classroom culture, in particular, on who students are invited to become in particular classrooms, and how they visibly and not-so-visibly engage in processes of resistance, compliance, or affiliation. Martin includes the *intrapersonal level* by examining students' views of what it means to know and do mathematics in the classroom, and the extent of their affiliation with learning mathematics. These multiple levels bring together discourses about what it means to be a mathematics student and what it means to be members of historically underserved communities, and how students individually participate in these discourses.

Various studies in mathematics education intersect Martin's framework in a number of places. For example, Boaler and Greeno (1998) take AP Calculus students and their classrooms as the focus of their analysis. For their part, they emphasise the relationship between the organisation of teaching in mathematics classrooms and students' developing relationships with mathematics. In their study, they interviewed groups of AP Calculus students from different classes who experienced different forms of instruction. The classrooms differed in the organisation of lessons and the relative openness of instructional tasks. Some were discussion-based while others followed more of a lecture-type format. In their analysis, they point to the relations between the classroom environments and students' views about mathematics and their own competence in mathematics. The majority of the students interviewed who were in the relatively open and discussion-based classes spoke favourably of their classroom experiences and their comments reflected more positive views of mathematics more generally. However, it is important to note, some students in the lecture-type classes also spoke positively of their experiences. The larger question that emerged relates to how the nature of instruction can influence not only the extent of students' learning, but also the nature of students' learning in terms of mathematical ideas and what it means to do and be successful in mathematics.

Sfard and Prusak (2005) make contact with Martin's (2000) framework at the intrapersonal level by drawing attention to students' narratives of their classroom experiences. As such, they treat identity as the stories that individuals and others tell about themselves. As they describe their perspective, stories are not seen as "windows" into identity. They are identity because they account for students' previous decisions and actions while providing direction for their current and future decisions.

The analysis presented draws on a conceptualisation of identity as narrative (Sfard & Prusak, 2005) that equates identity with the stories that individuals tell, in this case, about their classroom learning and the expectations that they must

meet in order to be successful. In this view, individuals' stories are seen as accounts on the state of affairs in their world. It is their understanding of how things are and how things should be. The value of such an approach lies in the relationship between individuals' stories and how they experience and move through different situations. One distinction in this approach is that stories are taken as individuals' views and are not analysed in comparison with how experiences "actually" transpired. The importance of stories lies in the relationship between these views and how individuals participate in different situations and communities. Individuals' narratives offer explanations for human functioning across experiences, in spite of criticisms that stories may not reflect the actual state of affairs. The central point is that stories are person-made, include individual and collective stories, and are imbued with messages that provide guidance for interpreting individual and collective experiences. Prior to sharing the collection of students' stories about their mathematical experiences, a background to the study including a description of the participants and school setting is presented.

Background to the Analysis

The analysis focuses on a group of eight second grade students who attended elementary school in a city in the Southeastern region of the United States. The school was located in what the teachers described as an affluent neighbourhood in the city. In addition, the student population was overwhelmingly Caucasian. The school had an enrolment in the elementary grades, kindergarten through fifth grade, of approximately 500 students.

The student participants of this study attended first grade in the same school and were all members of the same mathematics class. During that year, their teacher engaged in a project focusing on number relationships with mathematics education faculty from a local university. During this project, class sessions were video-recorded with two cameras. This first class of which the students were members can be described generally as an inquiry-based classroom (Cobb, Wood, Yackel, & McNeal, 1992). Upon entering the second grade, students in this first grade class were interspersed into three different classes. All three of these classes may be described as classrooms that reflect practices that are typical of mathematics classrooms in the United States (Stigler & Hiebert, 1999). In this analysis, I draw on interviews conducted with eight of the elementary students who participated in the project class about their first grade and second grade mathematical experiences.

Initially, I conducted observations of the three second grade classrooms for a period of six weeks. In addition, I spoke with the first grade teacher to determine the class that most reflected practices that characterised typical mathematics classrooms in the United States. The rationale for selecting this extreme case was to investigate students' experiences in classes that were as different instructionally as possible. The selected classroom also had the largest number of students from the first grade project class, which was eight students. Therefore the number and participants in the study were limited to this

particular group of eight students. I viewed the first month of videotaped class sessions from the first grade in order to establish the diversity of the students. In regards to racial diversity, only one student in the project class was considered a minority student and she was also a member of the second grade class that was selected for the study. In addition, I also consulted with both the first grade and second grade classroom teachers to ensure that the student participants reflected diversity in mathematics achievement and ways of participating verbally in class. In both the first grade and second grade teachers' opinions, the students reflected a range of abilities and understandings in mathematics. The group of eight was composed of three boys and five girls. One female student was African American with the remaining students Caucasian.

Data Sources and Analysis

The data for this analysis consist of audio taped individual interviews conducted with the eight students, videotaped lessons of their first grade class, and field notes of their second grade class. The author and a graduate student conducted the individual interviews during the fall semester of the academic year, during the elementary students' second grade year. Both researchers were present for all the individual interviews. Students were interviewed individually in a corner of the second grade hallway. The interviews focused on the students' thoughts about their mathematics lessons in each class, about their mathematical capabilities, and about their general impressions of doing mathematics in each class. Two separate interviews were conducted with each student, with the second yielding richer material. Students appeared to be apprehensive initially and the first interview served as a context for them to become familiar with the researcher. In designing the study, the focus was to understand differences in the mathematics class as experienced and observed by the students themselves. I chose to ask students about the first grade and the second grade classes separately, rather than asking them about differences outright, therefore emphasising that there indeed were differences between the two classes. My interest in the stories that students tell about their mathematics classes, about the roles they assume in their classes and about other students in their classes, encouraged us to conduct semi-structured interviews. This way, students were given prompts to elicit their comments, but they were provided with some freedom to voice what was important about what they experienced in each class. Special attention was paid to encouraging students to elaborate on their comments, as much as could be encouraged from seven and eight year old children. For example, students would use the term group time or class time as they made references to each class, however, it was clear that the same term had different meanings in the two classes. An effort was made to have students unpack these terms to uncover more accurately students' descriptions of their roles in each class.

I observed videotapes for twelve sessions of the first grade class and recorded field notes for these sessions. This was done in order to document the norms of participation and the classroom activity structure for the class.

Similarly, I observed first hand 12 sessions of the second grade class (two sessions per week) during the semester in which the individual interviews were conducted. These observations served to give direction to interview questions and to triangulate students' descriptions of the two classes in the individual interviews.

Data analysis procedures involved identifying and grouping segments related to the conjectured categories, and drawing on open coding (Strauss & Corbin, 1990) to develop possible themes that cut across interviews and individual students. The process of analysing interview data involved multiple phases of coding. I analyzed the interviews by first transcribing the audiotapes and analysing them student by student in order to identify an individual students' comments about their roles in each class, explicit difference in the two classes, and what it meant to be a good student in each class. This process involved analysing three interviews for one student, then examining another student's interviews, and so on and so forth.

After this initial stage, I looked across interviews for significant themes along the three categories of roles, differences, and competence that were mentioned in the above paragraph. I developed a chart that summarised each student's comments concerning the three strands in order to obtain a sense of commonality and variation across students. I then included examples of students' comments that illustrated these ideas. In this manner, conjectures about roles and competence in the views of the two classes were developed, tested, and revised as individual interviews were examined and relevant segments identified. An additional researcher analysed eight interviews, one from each student, to offer a different lens from which to examine the interview data and to offer alternative interpretations of students' comments. The coding results from these interviews were compared with those of the author. Differences, alternative interpretations, and ways of accounting for these differences were discussed and inter-rater reliability of 92% was calculated.

Another important aspect of data analysis was an examination of the field notes of sessions in the two classes. This aspect of the analysis served to provide background in order to develop interview questions and to consider along with student interview data for triangulation purposes. The field notes of class sessions were analysed in multiple phases in a manner similar to the student interviews (Glaser & Strauss, 1967). These phases included first working through field notes session by session in order to identify the classroom structure for each session and to identify central features of each class, including students' roles and expectations for competence. A second phase involved looking across class sessions in order to make conjectures about patterns of participation, aspects of student roles, and mathematical competence for each class, and to isolate sessions that illustrated significant aspects of life in each class. Particular attention was paid to situations that seemed to contradict ongoing conjectures and to offer explanations for these situations and how they informed the ongoing analysis.

Insights Gained

In this analysis, I first provide a description of mathematics lessons in the first grade and second grade classes, focusing on the organisation of lessons and students' obligations for participating in each class. Next, I describe the students' perceptions of these classes, drawing attention to differences in student roles and mathematical competence in each class. I present representative comments to provide a sense of the students' language and descriptions. Finally, I discuss the implications of the findings for mathematics teaching and learning and future research in mathematics education.

Mathematics Lessons in the Two Elementary Classes

The first grade class. "Ms. Brown's" first grade classroom reflected many characteristics of a reform-aligned classroom (NCTM Standards, 1989, 1991, 2000). Her classroom can be described in terms of the students and teacher engaging together in doing mathematics. A typical mathematics lesson began with the teacher posing a task for the students to investigate in smaller groups. These tasks involved a situation that provided a context for problem solving in which the students would engage. After the students had worked in small groups for approximately 15-20 minutes they returned to the large carpeted area in the classroom to participate in a whole-class discussion focusing on students' interpretations of the task. This whole-class discussion usually lasted 25-45 minutes. Therefore, the total time spent on mathematics daily in Ms Brown's class was 40 minutes to one hour.

The class discussion appeared to be the central part of mathematics lessons in the first grade class. While Ms Brown initiated and facilitated the topics that are addressed in the discussion, the students were active participants and did most of the talking. The discussion of students' responses usually began with Student A explaining how he or she thought about the problem. As an illustration, one of the tasks in which the class engaged as a group was to examine an arrangement of dots (called a dot flash) on an overhead projector for a brief moment, to explain how many dots they saw, and how they "saw" the dots. For example, one student saw a group of five dots as three and two, while another saw five individual dots. The purpose of this activity and others like it was to support students in developing a flexible understanding of number relationships. In most discussions of tasks, the teacher and students responded by asking questions to clarify and to understand Student A's way of thinking. There were explicit norms of participation in the classroom. For example, students were expected to listen carefully to each other and to ask each other questions when they did not understand. They were also expected to explain their solutions in such a way that their classmates would understand their methods and the rationale for these methods. Thus, merely stating a solution appeared to be unacceptable. For example, in the dot flash activity, students would state the numbers of dots and then describe how they saw the pattern of numbers. The class would also clarify the specific patterns that made up the dots,

as seen by the specific student. Few students offered only solutions in the class sessions that I observed, but when these students did they were asked about their reasoning and why they chose to perform certain procedures. It seemed typical for the teacher, once one student had presented his or her way, to ask for other ways to solve the problem. A typical discussion focused on only one, two, or three problems. Occasionally in the sessions that I observed, the teacher notated the students' methods on the board and sometimes the students notated their own approaches. It was also not uncommon to see two or more students at the board discussing a problem with reference to a picture. The following excerpt is a typical dialogue in this first grade class:

Teacher:	What were we trying to find out?
Patrick:	How many more bugs are in the box than in the jar?
Teacher:	How did you think about it?
Patrick:	I knew it was 15 because it shows that 15 is more than 9.
Student:	I don't understand Patrick's way.
Teacher:	Do you want to explain that to us Patrick?
Patrick:	I knew that since 15 was more than 9 it says how many more bugs are there in the box than the jar – Here it says 15 and in the jar it says 9 and that is how I knew.
Teacher:	Did anyone else think about the problem in a different way?
Paul:	I can help Patrick and then I have a different way.

In this classroom, the teacher's role involved creating problematic task situations for students and orchestrating mathematical discussions that provided students access to other students' ways of reasoning. The students' role related to understanding the ideas of others and clarifying their own thoughts by explaining and justifying their solution methods.

The second grade class. Mathematics lessons in the second grade class were quite different from the first grade class previously described. A typical mathematics lesson involved the students and teacher working together to complete pages from the textbook and accompanying workbook. The typical interaction during whole-class time was teacher-student interactions. The teacher, "Ms Smith," would typically ask a question, students would raise their hands, and the teacher would call on one student to respond. Sometimes Ms Smith asked students to read the problem, but predominantly, the teacher asked students for answers they had calculated to specific problems in their assignments. Occasionally, students were sent to the board, but only to write the answer to the problems. Most of the problems in their lessons involved decontextualised problems that required students to perform certain algorithms successfully. Students who raised their hands and answered a question were

correct the majority of the time. In one lesson I observed, only one student gave an incorrect solution out of the ten answers that were offered by different students. The problems appeared to have been routine for most of the students, even though the teacher indicated that the ideas were new to the students.

The typical mathematics lesson in this class was 20 to 40 minutes in duration and included a large number of problems in comparison to the first grade class, generally 15 or 20 from students' workbook or their textbook. Occasionally, the students worked in pairs, but the interactions in mathematics lessons typically involved one student talking with the teacher. The following excerpt of a lesson with base ten blocks is illustrative of a typical interaction in this class:

Teacher:	Work with your partner to show me 214 on your work mat. Now show me one less than 214. Phil, what is it now?
Phil:	213
Teacher:	How did you get that?
Phil:	I took one away from the ones.
Teacher:	Now put 214 back on your work mat, and take away 10.

Following this exchange, this particular lesson continued with students going through five or six problems that were quite similar to one less than 214. These examples were carried out in much the same way as the above exchange. The teacher stated what the students should do and the students provided the answers with no mention of methods. There were two situations in which students showed what they had done on their work mat, but again, the conversation was very procedural in nature, focusing on the steps rather than any kind of rationale for steps or how they saw the particular problems. As part of the class routine, students were asked regularly to complete work individually from their workbook. For the most part, the students in the class began working on these problems as Ms Smith circulated to examine their work. As this particular lesson illustrates, the teacher did present students with manipulatives to solve problems, in this case subtraction problems. However, even in these cases, lessons with manipulatives often focused on procedures to follow, with conversations discussing "moving things over" and "taking these things away".

Student roles in the second grade class consisted of the students raising their hands, providing solutions to problems presented in class, and being quiet while others were talking. The teacher's role was to direct the questions and to evaluate the answers, usually with praise as most were correct.

The Students' Perspective

In general, students' comments regarding their experiences in the two classes were quite consistent. Students' comments reflected a number of points about student roles and mathematical competence. The prominent themes involved how students' roles involved sharing their "way" in one class and following the

teacher's directions in the second. Mathematical competence was also described in terms of a student using her way to solve a problem and of explaining her rationale to the teacher and fellow classmates. In the second class, the criteria for success involved how well students repeated steps in a particular sequence. I unpack what student roles and competence entailed in each of the classes further at a later point. Prior to doing so, I first make note of one finding, of how all the students experienced success in both of their classes, as reflected in their comments. Following this, I explore the differences in *why* the students felt competent in each class through a discussion of roles and of the criteria for competence.

Students' views of their own competence. All the students interviewed indicated that they felt confident and capable in doing mathematics in both their first grade and second grade classes. When asked if they were good mathematics students last year, all the students remarked that they were. Five of them indicated that they did a good job or they did what they were supposed to do in class. One student remarked that she was because the teacher would tell the class that they were "all good in maths". In regards to their second grade year, all the students also commented that they felt they were good students. They again commented with phrases such as "I follow directions" or "I do what I'm supposed to in class." Two students commented that they did a "good job" because Ms Smith always gave them "second chances" and "it was okay to make a mistake". Students' comments relating to their own feelings of competence reflected positive comments and a sense that students felt both classroom environments were supportive and understanding in light of their mathematical learning.

Student roles in the mathematics lessons. One of the main features of the first grade class that came out of student interviews was the time that students spent sitting on a large rug in the classroom engaging in whole-class discussions. This seemed to be a memorable activity, as indicated by students. All the students mentioned the whole-class discussion time in their descriptions of "what maths was like" in the first grade class. All but two of the students gave comments that described the discussions in positive terms. One student indicated that the focus of the discussions was talking about how you did the problem. Mark made the following comments:

> You had to talk about your way and let everybody help you think about it. Sometimes, everybody had to ask questions about the thing somebody said. That's okay. You had to listen really hard.

Another student indicated that discussions were productive for her, and in doing so touched on discussions as learning opportunities. Mary described:

> I learned about stuff, like the numbers to ten when we all sat down to talk at the front. I kinda didn't like it, but then I did because nobody laughed or said it was wrong.

Another student described the discussions as sharing each other's ways. Consider Phil's comments in the following excerpt.

Interviewer: After you explained your answer and you got the right answer, do you go on to another problem? I'm talking about this year.

Phil: It's time to go on when we get the right answer [second grade class]. Last year if you got the right answer, Ms Brown would say, 'Yes'. Then she would ask, 'Who did it another way?' And if you had a different way you would tell 'em how you did it.

All the students in the class described their first grade teacher as asking "a lot of questions about your way." One theme in the students' comments was how the teacher was interested in how the students thought about mathematics problems. As one student, Anna, noted,

> She talks to us a lot. She asks questions, not about the problem. She wants to know what we think about it.

This idea of wanting to know about students' interpretations of problems was reflected in another student's comments. Patrick describes,

> She wants to know how you did it…your idea in the problem.

Students also indicated that in the first grade, the teacher would not "tell the right answer" at the outset. One student named Phil remarked, "Ms Brown didn't tell us the answer…she waited."

Interviewer: She waited?

Phil: She listened to me even if my answer was wrong.

Interviewer: Did you like that?

Phil: I liked to tell why I did it…not just my answer.

Another student echoed Mark's sentiments about explaining more than the answer to mathematics problems. Paige commented, "I like to tell my answer, but I like to talk about it more. That's what we did in Ms Brown's class."

Interviewer: What do you mean talk about it more?

Paige: How I got the answer…that's what Ms Brown wanted to know.

Interviewer: Why did Ms Brown want to know about that?

Paige: Hmmm…cause she said she wanted to know about how to do it.

Paige's comments reflect the view that the teacher valued and learned from the students' sharing of their problem interpretations. Her comments further emphasise the role of whole-class discussion time as a central part of the organisations of mathematics lessons in the first grade class.

In further describing their first grade class, five of the students mentioned that listening was an important part of class time. This involved listening to both the teacher and other classmates. As an illustration:

Mary: She [the teacher] said we should listen to her and to our friends because we might have questions about what people did, not just what the teacher did.

Interviewer: Is that important, to listen to the teacher and other people in class?

Mary: She [Ms Brown] said that we learn from her and our friends, not just her.

This emphasis on discussions was in contrast to the prominent feature that students described in their second grade class, that of doing seatwork, individually or in small groups. All but three of the students mentioned seatwork as the primary activity in which they engaged during mathematics lessons. The following comments are representative.

Patrick: The main thing we do is do the problems…we do it alone or with somebody, but we have to do the work. Then the teacher tells us the answers.

Amy: We do mathematics at our desks. She tells us a little and we figure it out.

Interviewer: Then what?

Amy: Then she tells us some more about it and then she looks at the number we got.

Students also described the aspects of their roles during "group time" in their second grade class. Taken collectively, students' comments indicated that group time consisted of a time when they shared their answers, checked their answers, and learned about new kinds of problems in mathematics. Seven out of the eight students noted that they sometimes provided answers to the problems and explained their steps only if the answer was incorrect. Five of these seven also commented that the teacher explained their mistakes in a supportive way. For example:

Paige: We give the number we got in group if she calls our name. She keeps going till we get the right answer. Most of the time I get it right. If I don't I tell what I did and she tells where I should go back to. She helps me do it the right way.

Four of the eight students remarked that during this group time the teacher focused on the particular procedures for solving problems in mathematics lessons. One student, Patrick, commented:

> She tells us about counting up to add 'cause some people forgot that I do that…she says it's the easiest way to do it.

In a similar vein, one student, Mary indicated the importance of the steps:

> She tells us step by step, that's what she says so we won't mess up the answer.

Two students emphasised the importance of listening carefully to the teacher in group time so they would know what to do during individual work. As an illustration, Phil indicated, "You have to be a good listener so you know the steps to do for the mathematics work."

All the students except one described another way group time sessions were organised. This involved the teacher moving from one student to another until arriving at a correct answer; then, she would explain the steps at that point to clarify incorrect answers. The following comments by Mark and Alice are representative of this idea.

> Mark: She [the second grade teacher] wants to know the number we got. She tells us what to do and she wants to see if we got it right.
>
> Alice: We check the work.
>
> Interviewer: How do you check it?
>
> Alice: I say the answer and she stops if it's the right one.
>
> Interviewer: If it's not the right one, what does she do?
>
> Alice: She keeps going. She calls on someone else and they tell her the answer they got.

Although these students' comments also reflect the teacher's role in the classroom, they clarify the obligations that students would have to fulfill to be successful in the second grade class, during seatwork and group time in the mathematics lessons.

What it means to be a good student in mathematics class. When asked why they thought they were good mathematics students in each class, students gave markedly different reasons. As stated above, students initially gave very general responses to why they were good students; prompting interviewers to ask many follow up questions in order to seek some specificity. The differences in why they were competent in each class centred on what doing mathematics came to mean in each class with the first grade class involving talking thinking, and listening, as compared to the focus on steps and answers for the second grade class. The following comments provide illustrations of these differences.

> Patrick: I do the problems like Ms S has told us [in the first grade class].
>
> Patrick: I thought about it and told what I thought [in the second grade class].

Another student, Amy, noted similar differences in what it meant to be a good student in the two classes.

> Amy: It's easy because I remember stuff Ms S said about it [in the first grade class].
>
> Amy: I had to talk and listen hard to other people. I did it even if I was tired [in the second grade class].

All of the students at one point commented on a specific slice of competence within each of the classes. Three of the students noted that they were good students because they listened to their friends. As one student remarked,

> I listen to other people's ways. I did a good job because then I could ask questions.

Four of the students specifically used the verb "see" as they talked about their participation in the first grade class. Consider the following students' comments as an illustration of this.

> Mary: I got good at seeing the problem in my mind.
>
> Patrick: You have to look at it and wait 'til you see something.

Three students also noted that you had to share something or ask questions as a way to participate in the discussions. For example, Tim commented, "I ask questions. You can say your way, but I didn't like my way all the time. I just asked questions and that was good."

In students' descriptions, the first grade teacher was included sparingly. One student noted how she "made sure everything was fair". Another student shared that Ms Brown "checked to give everyone a chance". The students' descriptions of the criteria for success in the first grade class focused on students' participation in discussions. The nature of this participation involved thinking about problems presented to them, sharing their way, and asking questions of other students. The teacher's role in determining their success had to do with her organisation of discussions and giving "everyone a chance," as indicated by students' comments.

In regards to the second grade class, the majority of students' comments were also positive concerning their own competence and the teacher's role in supporting their competence. Seven of the students indicated that it was important for them to listen carefully to the teacher's explanation of steps to solve problems. For example, Mary commented:

> Ms Smith lets you do it some other ways, but she tells one way. That way is the best so I listen to know the steps. That's why I do good in there.

Four of the students also offered that remembering the steps was critical to their success in the class. Consider the following illustration:

Alice: I just remember what she says when I do it. I remember it so I do it the same way. Then I know it's right. I don't have to guess.

Students' explanations related to their own competence in class often included references to the teacher, as contrasted with the explanations related to their first grade class. There was not clear consensus from their comments as to the teacher's exact role in supporting their competence. One or two students pointed out a dimension of the teacher's role. For example, one student commented on the role of the teacher in determining the correctness of answers. Mary commented: "I do the maths work and its right most of the time…"

Interviewer: Do you know when you've done it right?

Mary: I'm not sure. I wait for Ms Smith to check it or tell us the answer.

Interviewer: Are you a good student in there?

Mary: Yeah, Ms Smith says my answer is right a lot of times.

Two students also indicated that the second grade teacher let you know if how you solved the problem, if it was not "her way," was also another effective way. Students' comments focused on the teacher determining if their methods made sense. One student addressed this topic.

Phil: I do it a different way and she says everybody's is a good way. She tells us if it gives us the same answer, you know, the right answer.

Phil's comments conveyed that to him the teacher evaluated other methods positively, but she stated whether or not that particular method would yield a right answer.

In the first grade class, being a good student involved thinking about mathematical ideas and communicating these thoughts to the class, as indicated by students' comments. All but one of the students described competence in these different ways. This one student noted that being a good mathematics student meant following directions and behaving as one should in class, reflecting the general rules of the class. For most of the students, being a good student in the second grade class entailed following the teacher's directions in solving problems, remembering these procedures, and being patient. Students' comments reflected differences in both the criteria for competence and the teacher's roles in supporting their competence in each class.

Discussion

This analysis has presented a group of children who experienced mathematics positively as they were members of two classes in which mathematics was taught very differently. For this reason, the analysis has focused more on the students' explanations of their success in each of the classes. The students' comments

about the classes were quite consistent with my own observations of the two classes. Initially, I had anticipated that one limitation of the study would be that students would have difficulties in recalling the previous academic year, their first grade year. Although there were undoubtedly details that they did not recall about their mathematics lessons, students seemed to reflect easily on the central features of their first grade class. The number of participants was also a limitation of the study, and it is quite feasible that students' comments were influenced, at least initially, by the researcher's presence and emphasis in the interviews on differences between the two classrooms.

In spite of these limitations, I believe that there are significant points to be gleaned from the students' perspective in this analysis. First, although students experienced feelings of success and competence in both classes, how students described their roles in each class was quite different. In addition, what it meant to do mathematics in each class was different, as reflected in students' comments. As mathematics educators we strive to support all students' positive sense of self and their identification with mathematics learning. At the same time, it is critical to consider what students are identifying with as they participate and learn in classrooms. This involves understanding how students think about what it means to learn and do mathematics, not only their feelings of competence as they engage in this process. In this analysis, with children, it is difficult to gain a sense of the extent of students' affiliation with mathematics as compared to compliance with the expectations of the teacher's spoken and unspoken "rules" of the class. Further, this was not the intent of the analysis. It is difficult to determine at this point if these students are on trajectories in which they position themselves in ways similar to those students in the study of Boaler and Greeno (2000) who described themselves as competent in AP Calculus, but admitted to being disconnected with mathematics and noted that they would not choose to study mathematics further, in spite of their success in the subject throughout their academic careers. Or, are the children in this analysis on paths that include identification and mastery in the long term? Again, it is difficult to say. Perhaps the larger point is the consideration of students' experiences of competence and what that competence entails within a classroom and what mathematical competence comes to mean over time, as constituted in students' participation across classrooms and across multiple communities in school and out of school. This consideration focuses attention on students' mathematical content ideas and what students think as they are doing certain kinds of mathematics. Mathematical literacy can then include development of understanding, skills, and expansive views of what mathematical competency entails.

The second point relates to aspects that contributed to students' experiences of competence in both classes. Clearly, both classroom cultures were contexts in which students could display and develop their sense of competence. Perhaps these feelings of competence arose from both teachers' abilities to offer explicit explanations for procedures and expectations and to cultivate positive classroom environments that appreciated individual differences and were understanding of mistakes. One avenue of conjectures is to consider the classroom including

pedagogy, curriculum, and any other resources on which the teacher might have drawn for instruction. As the work of Martin (2000) indicates, the classroom is only one community in which one might seek explanations to how students' ideas about mathematics and about themselves as mathematics learners. As his work indicates, it is a worthwhile endeavour to understand the broader contexts in which students participate in practices that contribute to how they make sense of their roles in class and while they are learning mathematics. These broader sites include racial groups, students' peer groups, families, and more local communities with which students identify.

As a final point, this analysis is a comparison of two classrooms in which instruction was organised differently in significant ways. Some might read this article as a comparison of reform-based instruction to more traditionally-designed instruction. Stepping back, the analysis indicates some direction for future research by highlighting the relationship between how teaching is organised and students' thinking about their roles and what it means to be a mathematics student, as it is constituted in a particular class. The construct of identity as narrative emphasised a focus on students' stories about their place in, and understanding of, learning mathematics in each class. This focus was centered on roles and competence that developed through aspects of the classroom. Future research is needed to clarify classroom and other student community aspects that contribute to students' ideas about their roles, competence, and what it means to learn mathematics in school. Such research is critical in order to understand better long-term issues of persistence and achievement in mathematics for all students.

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